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Knowledge and Strategic Ability for Model Checking: A Refined Approach

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Knowledge and Strategic Ability for Model Checking: A Refined Approach

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Abstract

We present a translation that reduces epistemic operators to strategic operators in the context of model checking. The translation is a refinement of the one from [7, 9], and it improves on the previous scheme in two ways. First, it does not suffer any blowup in the length of formulae (the one from [7, 9] did). Second, the new translation is defined in a more general setting: additional constraints can be imposed on strategy profiles that agents can execute; we show that the translation is still valid in such a general case.

1 Introduction

Modal logics of multi-agent systems usually combine several dimensions. Knowledge, time, actions, strategic abilities, norms/obligations, intentions, desires etc. can all be involved in a description of an agent system. This way, modal logic can support sufficiently realistic descriptions of agents. But there is a price to pay: such multi-modal logics are usually harder to handle semantically as well as algorithmically. Thus, a designer is usually faced with the task of finding a good tradeoff between a “clean” logic with few modalities (and clear overall semantics) and a "realistic" language with many modalities (where it is not immediately visible how parts of the semantic interfere). A reduction method that allows to express one modality with the others offers two kinds of advantage. In terms of theory, it allows to make the logic “cleaner”, and study its theoretical properties (semantics, computational complexity) in a simpler environment. On the practical side, we can reuse the advances in, say, model checking of one sort of modality to improve the techniques used for dealing with the other dimensions.

In [7, 9], we proposed how epistemic modalities can be equivalently expressed by strategic operators in the context of model checking. Formally, it was done by a reduction of ATEL model checking to ATL model checking.

The reduction was polynomial in *almost* every respect. Unfortunately, the length of formulae *could* suffer exponential blowup (although the number of *different* subformulae in the formula could increase only linearly). We argued that, for most model checking algorithms, it would not increase the verification time. Still, it is a flaw that makes using the reduction awkward, at least for theoretical purposes. The aim of this Technical Report is to propose a slight update of the reduction that does not suffer from the blowup any more. Moreover, we point out that the reduction can be used even if we impose some "behavioral constraints" on the strategies that can be played by agents. Thus, the method can be used also for variants of ATL where we assume that the agents can only play in a uniform [12], socially acceptable [18], or rational way [10].

Our presentation here is based on some material from [7, 9]. It should be also mentioned that the original reduction was inspired by [16], and shared some similarities with [20] (although the reduction proposed in the latter paper had a much more limited scope). Our presentation of strategic constraints is based on the approach of [10].

2 What Agents Can Achieve

ATL is a logic that allows to reason about what agents can achieve in game-like scenarios. As it does not include imperfect information in its scope, it can be seen as a logic for reasoning about agents who always have complete knowledge about the current state of affairs.

2.1 ATL: Ability in Perfect Information Games

ATL [1, 2] can be understood as a generalization of the branching time temporal logic CTL [3, 4], in which path quantifiers are replaced with so called *cooperation modalities*. The formula $\langle\langle A\rangle\rangle\varphi$, where A is a coalition of agents, expresses that A have a collective strategy to enforce φ. ATL formulae include temporal operators: "$\bigcirc$" ("in the next state"), $\Box$ ("always from now on") and $\mathcal{U}$ ("until"). Operator $\Diamond$ ("now or sometime in the future") can be defined as $\Diamond\varphi \equiv \top\mathcal{U}\varphi$. Similarly to CTL, every occurrence of a temporal operator is immediately preceded by exactly one cooperation modality.[1] The broader language of ATL*, in which no such restriction is imposed, is not discussed in this paper.

Formally, the recursive definition of ATL formulae is:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A\rangle\rangle \bigcirc \varphi \mid \langle\langle A\rangle\rangle \Box\varphi \mid \langle\langle A\rangle\rangle \varphi\,\mathcal{U}\,\varphi$$

[1] The logic to which such a syntactic restriction applies is sometimes called *"vanilla"* ATL (resp. "vanilla" CTL etc.).

where A is a set of agents. Example ATL properties are: $\langle\langle jamesbond\rangle\rangle\Diamond\mathsf{win}$ (James Bond has an infallible plan to eventually win), and $\langle\langle jamesbond, bondsgirl\rangle\rangle\mathsf{fun}\,\mathcal{U}\,\mathsf{shot}$ (Bond and his current girlfriend have a collective way of having fun until someone shoots at them).

A number of semantics have been defined for ATL, most of them equivalent [6, 7]. In this paper, we use a variant of *concurrent game structures* as models. A *concurrent game structure* (CGS) is a tuple $M = \langle \mathbb{A}\mathrm{gt}, St, \Pi, \pi, Act, d, o\rangle$ which includes a nonempty finite set of all agents $\mathbb{A}\mathrm{gt} = \{1,\dots,k\}$, a nonempty set of states St, a set of atomic propositions Π, a valuation of propositions $\pi : St \to \mathcal{P}(\Pi)$, and a set of (atomic) actions Act. Function $d : \mathbb{A}\mathrm{gt} \times St \to (\mathcal{P}(Act)\setminus\emptyset)$ defines nonempty sets of actions available to agents at each state, and o is a (deterministic) transition function that assigns the outcome state $q' = o(q, \alpha_1, \dots, \alpha_k)$ to state q and a tuple of actions $\langle\alpha_1,\dots,\alpha_k\rangle$, $\alpha_i \in d(i,q)$, that can be executed by $\mathbb{A}\mathrm{gt}$ in q.

A (memoryless) *strategy* s_a of agent a is a conditional plan that specifies what a is going to do for every possible situation: $s_a : St \to Act$ such that $s_a(q) \in d(a,q)$. We denote the set of such functions by Σ_a. A *collective strategy* s_A for a group of agents A is a tuple of strategies, one per agent from A; the set of A's collective strategies is given by $\Sigma_A = \prod_{a\in A}\Sigma_a$. The set of all *strategy profiles* is given by $\Sigma = \Sigma_{\mathbb{A}\mathrm{gt}}$.

A *path* λ in model M is an infinite sequence of states that can be effected by subsequent transitions, and refers to a possible course of action (or a possible computation) that may occur in the system; by $\lambda[i]$, we denote the ith position on path λ. Λ_M denotes all paths in model M. The set of all paths starting from q is given by $\Lambda_M(q)$. The subscripts will be omitted when the model is clear from the context.

Function $out(q, s_A)$ returns the set of all paths that may result from agents A executing strategy s_A from state q onward. Let $s_A(a)$ denote agent a's part of the collective strategy s_A.:

$out(q, s_A) = \{\lambda \in \Lambda(q) \mid$ for every $i = 1, 2, \dots$ there exists a tuple of agents' decisions $\langle\alpha_1,\dots,\alpha_k\rangle$ such that $\alpha_a = s_A(a)(\lambda[i-1])$ for each $a \in A$, and $\alpha_a \in d(a, \lambda[i-1])$ for each $a \notin A$, and $o(\lambda[i-1], \alpha_1, \dots, \alpha_k) = \lambda[i]\}$.

Formally, the semantics of ATL formulae can be given via the following clauses:

$M, q \models p$ iff $p \in \pi(q)$ (for $p \in \Pi$);

$M, q \models \neg\varphi$ iff $M, q \not\models \varphi$;

$M, q \models \varphi \wedge \psi$ iff $M, q \models \varphi$ and $M, q \models \psi$;

$M, q \models \langle\langle A\rangle\rangle \bigcirc \varphi$ iff there is a collective strategy s_A such that, for every $\lambda \in out(q, s_A)$, we have $M, \lambda[1] \models \varphi$;

$M, q \models \langle\langle A\rangle\rangle \Box \varphi$ iff there exists s_A such that, for every $\lambda \in out(q, s_A)$, we have $M, \lambda[i] \models \varphi$ for every $i \geq 0$;

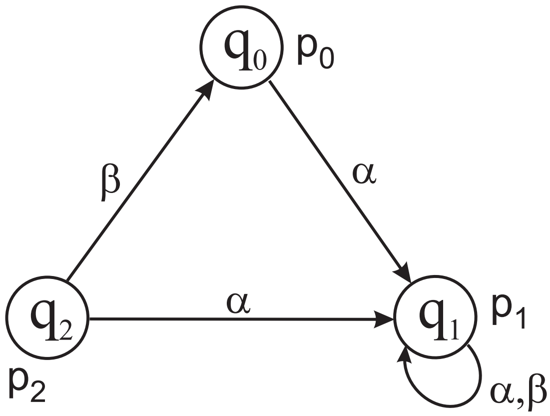

Figure 1: Simple concurrent game structure

$M, q \models \langle\langle A \rangle\rangle \varphi \mathcal{U} \psi$ iff there exists s_A such that for every $\lambda \in out(q, s_A)$ there is an $i \geq 0$, for which $M, \lambda[i] \models \psi$, and $M, \lambda[j] \models \varphi$ for every $0 \leq j < i$.

Example 1 *Consider a very simple concurrent game structure M_1, shown in Figure 1. There is only a single agent (a) in the model. Example* ATL *formulae that hold in states of the model are:* $M_1, q_1 \models \mathsf{p}_1$, $M_1, q_2 \models \langle\langle a \rangle\rangle \bigcirc \mathsf{p}_1$, $M_1, q_2 \models \neg\langle\langle \emptyset \rangle\rangle \bigcirc \mathsf{p}_1$, *and* $M_1, q_2 \models \langle\langle \emptyset \rangle\rangle \Diamond \mathsf{p}_1$.

It is worth noting that the path quantifiers A, E of CTL can be expressed in ATL with $\langle\langle \emptyset \rangle\rangle$, $\langle\langle \mathbb{A}\mathrm{gt} \rangle\rangle$ respectively.

2.2 ATL with Epistemic Logic

Real-life agents seldom possess complete information about the current state of the world. On the other hand, imperfect information and knowledge are handled in epistemic logic in a natural way. A combination of ATL and epistemic logic, called *Alternating-time Temporal Epistemic Logic* (ATEL) was introduced to enable reasoning about agents acting under imperfect information [19].

ATEL enriches the picture with an epistemic component, adding to ATL operators for representing agents' knowledge: $K_a\varphi$ reads as "agent a knows that φ". Additional operators $E_A\varphi$, $C_A\varphi$, and $D_A\varphi$, where A is a set of agents, refer to *mutual knowledge* ("everybody knows"), *common knowledge*, and *distributed knowledge* among the agents from A. Models for ATEL extend concurrent game structures with epistemic accessibility relations $\sim_1, \ldots, \sim_k \subseteq Q \times Q$ (one per agent) for modeling agents' uncertainty.[2] We call such models *concurrent*

[2] The relations are assumed to be equivalences.

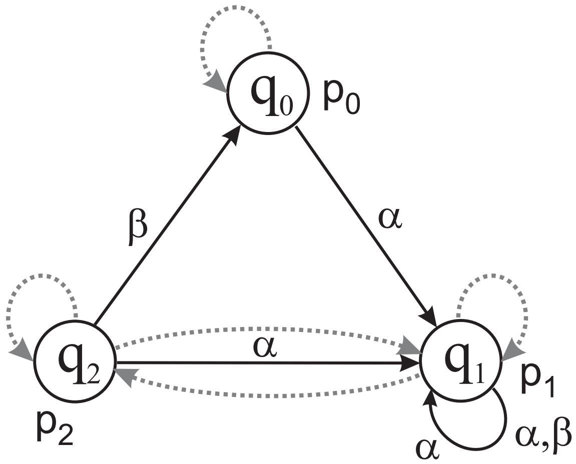

Figure 2: Simple concurrent epistemic game structure M_2

epistemic game structures (CEGS).[3] Agent a's epistemic relation is meant to encode a's inability to distinguish between the (global) system states: $q \sim_a q'$ means that, while the system is in state q, agent a cannot determine whether it is in q or q'. Then, the semantics of K_a is defined as:

$M, q \models K_a\varphi$ iff $M, q' \models \varphi$ for every q' such that $q \sim_a q'$.

Relations $\sim_A^E$, $\sim_A^C$ and $\sim_A^D$, used to model group epistemics, are derived from the individual relations of agents from A. First, $\sim_A^E$ is the union of relations $\sim_a$, $a \in A$. Next, $\sim_A^C$ is defined as the transitive closure of $\sim_A^E$. Finally, $\sim_A^D$ is the intersection of all the $\sim_a$, $a \in A$. The semantics of group knowledge can be defined as below (for $\mathcal{K} = C, E, D$):

$M, q \models \mathcal{K}_A\varphi$ iff $M, q' \models \varphi$ for every q' such that $q \sim_A^{\mathcal{K}} q'$.

Note that $K_a \equiv C_{\{a\}} \equiv E_{\{a\}} \equiv D_{\{a\}}$, so individual knowledge operators K_a are in fact redundant.

Example 2 *Let us extend the* CGS *from Example 1 by adding some epistemic links as shown in Figure 2. Example* ATEL *formulae that hold in states of the* CEGS *are:* $M_2, q_1 \models \mathsf{p}_1$*;* $M_1, q_1 \models K_a \neg \mathsf{p}_0$*;* $M_1, q_2 \models \neg\langle\langle\emptyset\rangle\rangle \bigcirc \mathsf{p}_1$*; and* $M_1, q_2 \models \langle\langle\emptyset\rangle\rangle \Diamond \mathsf{p}_1$.

It has been observed in several places that the meaning of ATEL formulae can be counterintuitive [8, 12, 13]. Most importantly, one would expect that an agent's ability to achieve property φ should imply that the agent has

[3] Additionally, we will assume that CEGS are *uniform*, i.e., agents have the choices in indistinguishable states ($q \sim_a q'$ implies $d_a(q) = d_a(q')$).

enough control and knowledge to *identify* and *execute* a strategy that enforces φ (cf. also [17]). ATEL does not preserve these requirements, as it implicitly assumes that the layers of ability and knowledge are independent. We do not deal with the issue explicitly here (an interested reader is referred to [12, 11] for an extensive discussion). We point out, however, that the inclusion of strategic constraints, presented in Section 3, allows e.g. to impose that the agents use only uniform (executable) strategies.

3 Restricting Strategies of Agents

In many cases, it seems appropriate to put some constraints on the "good" (allowed, legal etc.) behaviors. For instance, in scenarios with imperfect information we may consider only *uniform* strategies, i.e., ones that specify same choices in indistinguishable states. Or, we may assume that agents are only going to play strategy profiles that are in Nash equilibrium. We define a class of such *strategic constraints* in this section, with the aim of showing that our translation works well also for scenarios where agents' choices are additionally restricted.

Our constraints are based on the idea of plausibility sets [10], and generalize the behavioral constraints from the framework of social laws [18].

3.1 Strategic Constraints

A behavioral constraint in [18] is a function $\beta : \mathbb{A}\mathrm{gt} \times St \rightarrow \mathcal{P}(Act)$ that specifies which actions can be "legally" played by agents. More specifically, $\beta(a, q)$ is the set of actions that a is allowed to play at state q. Naturally, $\beta(a, q) \subseteq d(a, q)$, and the inclusion may be strict. $\beta(a, q)$ is assumed to implement a social norm: agent a (when in state q) may be forbidden to play some actions in his repertoire; if he decides to play them, he will violate the norm.

Note that using constraints of this type implies that norms can only apply to actions of individual agents (independently). It is therefore not possible to specify e.g. that one is allowed to shoot in self-defense, i.e., *right at the moment* when another person is trying to harm him. Likewise, norms of that type specify legal actions independently for each state. Thus, if we do not accept lying, then making a false statement will be always forbidden, even if it is just a joke, and the speaker is going to disclose the truth in the very next moment.

Here, we are looking for a model that enables to cope with such interrelationships between the allowed actions of different agents at different states, too. Another rationale for this comes from game theory. Unlike in normative systems, we are interested in "rational" rather than "moral" behavior there, but the general pattern is the same. That is, some strategy profiles of agents (e.g., those in Nash equilibrium) are deemed "rational", while the others are rejected as "irrational". Note that, especially for Nash equilibrium, the rationality of an action does depend on what the agent is going to do at other states; even more: it also depends on what *the other agents* are going to do at this and other states. Thus, our requirements with respect to agents' behavior will be modeled as *sets of strategy profiles*.

When defining agents' behavior via strategy sets, one assumes implicitly that agents actually *play strategies*. In our case, it would for instance imply that each agent does the same action every time the system comes back again to one of the previous states (as memoryless strategies are used in our semantics of ATL). This is a very strong assumption, and we do not always want to make it with respect to *all* agents. Thus, our strategic constraints will also include the set of agents to whom the constraint should apply.

Definition 1 *A* strategic constraint *is a pair $\eta = \langle \Upsilon, A \rangle$, where $\Upsilon \subseteq \Sigma$ is a non-empty set of strategy profiles and $A \subseteq \mathbb{A}\mathrm{gt}$ is a set of agents.*

Now we can define what it means for a strategy to be consistent with a strategic constraint, and what is the outcome of a strategy under a strategic constraint.

Definition 2 (Substrategy) *Let $A, B \subseteq \mathbb{A}\mathrm{gt}$, and let s_A be a collective strategy for A. We use $s_A[B]$ to denote the* substrategy *of s_A for agents from B only, i.e., strategy $t_{A\cap B}$ such that $t^a_{A\cap B} = s^a_A$ for every $a \in A \cap B$. We extend the notation to sets in a natural way: for a set of collective strategies $\Upsilon_A \subseteq \Sigma_A$, we define $\Upsilon_A[B] = \{t \in \Sigma_{A\cap B} \mid \exists s_A \in \Upsilon_A . t = s_A[B]\}$.*

Definition 3 (Consistency with a constraint) *Let s_A be a collective strategy of $A \subseteq \mathbb{A}\mathrm{gt}$, and $\eta = \langle \Upsilon, B \rangle$ be a strategic constraint. Strategy s_A is* consistent with constraint *η iff the part of s_A to which the constraint should apply occurs in Υ, i.e., $s_A[B] \in \Upsilon[A \cap B]$.*

Additionally, for a strategic constraint η, we use $\eta(s_A)$ to denote all strategy profiles that are consistent with η and contain s_A as substrategy.

Definition 4 (Outcome under constraint) *Let M be a CGS, and q a state in M. Furthermore, let s_A be a collective strategy, and $\eta = \langle \Upsilon, B \rangle$ be a strategic constraint. The outcome of s_A from q under constraint η contains all paths which may result from agents A executing s_A from q on, when only strategies complying to η can be played by the opponents. As the constraint applies only to players from B, and the strategy for A is given, η restricts in fact only choices of $B \setminus A$. Formally, the set is defined as:*

$out(q, s_A, \eta) = \{\lambda \in \Lambda(q) \mid$ *there is* $t \in \Sigma_{A\cup B}$, *consistent with* η, *such that* $t[A] = s_A$ *and for every* $i = 1, 2, \ldots$ *there exists a tuple of agents' decisions* $\langle \alpha_1, \ldots, \alpha_k \rangle$ *for which:* $\alpha_a = t^a(\lambda[i-1])$ *for* $a \in A \cup B$, $\alpha_a \in d(a, \lambda[i-1])$ *for* $a \notin A \cup B$, *and* $o(\lambda[i-1], \alpha_1, \ldots, \alpha_k) = \lambda[i]\}$.

The following properties are immediate.

Proposition 1 $out(q, s_A, \eta) = \emptyset$ *iff* s_A *is inconsistent with* η.

Proposition 2 *For every* s_A, Υ*, we have that* s_A *is consistent with* $\langle \Upsilon, \emptyset \rangle$.

Proposition 3 *Let* $\eta = \langle \Upsilon, B \rangle$. *Then:*

$$out(q, s_A, \eta) = \bigcup_{t \in \eta(s_A)[A \cup B]} out(q, t).$$

The last property is an immediate corollary of Proposition 3:

Proposition 4 *For every* q, s_A, Υ*, we have that* $out(q, s_A) = out(q, s_A, \langle \Upsilon, \emptyset \rangle)$.

3.2 Abilities under Strategic Constraints: Semantics

The intuition behind strategic constraints is rather simple: for a constraint $\eta = \langle \Upsilon, B \rangle$ we assume that the actual collective strategy of agents B must occur somewhere in Υ. Note that the agents from B do not have to be all in the same coalition – B can collect both "proponents" and "opponents". The formal semantics of ATL formulae in the presence of strategic constraints is given by the clauses below.

$M, q, \eta \models p$ iff $p \in \pi(q)$ (for $p \in \Pi$);

$M, q, \eta \models \neg\varphi$ iff $M, q, \eta \not\models \varphi$;

$M, q, \eta \models \varphi \wedge \psi$ iff $M, q, \eta \models \varphi$ and $M, q, \eta \models \psi$;

$M, q, \eta \models \langle\langle A \rangle\rangle \bigcirc \varphi$ iff there is a collective strategy s_A, consistent with η, such that for every $\lambda \in out(q, s_A, \eta)$ we have $M, \lambda[1], \eta \models \varphi$;

$M, q, \eta \models \langle\langle A \rangle\rangle \Box \varphi$ iff there exists s_A consistent with η, such that for every $\lambda \in out(q, s_A, \eta)$ we have $M, \lambda[i], \eta \models \varphi$ for every $i \geq 0$;

$M, q, \eta \models \langle\langle A \rangle\rangle \varphi \mathcal{U} \psi$ iff there exists s_A consistent with η, such that for every $\lambda \in out(q, s_A, \eta)$ there is an $i \geq 0$, for which $M, \lambda[i], \eta \models \psi$, and $M, \lambda[j], \eta \models \varphi$ for every $0 \leq j < i$.

Consider model M_1 from Example 1, and let us assume that $\mathsf{p}_0 \equiv \mathsf{p}_1 \equiv \mathsf{lose}$, and $\mathsf{p}_1 \equiv \mathsf{win}$. Moreover, we assume that a rational agent would like to win as quickly (and as often) as possible. The following strategic constraint captures the assumption that the agent from M_1 is rational: $\eta = \langle \{[q_0 \mapsto \alpha, q_1 \mapsto \alpha, q_2 \mapsto \alpha]\}, \{a\} \rangle$, where $[q_0 \mapsto \alpha, q_1 \mapsto \alpha, q_2 \mapsto \alpha]$ is the strategy that specifies that a plays α in states q_0, q_1 and q_2.[4] Then, we have for instance that $M_1, q, \eta \models \langle\langle \emptyset \rangle\rangle \bigcirc \mathsf{win}$

[4] Note that $\Sigma = \Sigma_a$ in M_1 since there is only one agent in that model.

for each state q: a rational agent must win immediately in the next moment, regardless of the initial position.

A useful example of strategic constraints are so called *uniform strategies*. We mentioned in Section 2.2 that a realistic notion of ability under imperfect information should only refer to executable strategies. Uniform strategies capture this requirement in a simple way. We say that s_a is uniform iff, for every q, q', $q \sim_a q'$ implies that $s_a(q) = s_a(q')$; that is, agent a must specify same choices in states that look the same to him. A collective strategy s_A is uniform iff it consists only of uniform individual strategies. Let Σ_a^u denote the set of uniform strategies of agent a. Then $\Sigma_A^u = \prod_{a \in A} \Sigma_a^u$ is the set of collective uniform strategies of A, and $\Sigma^u = \Sigma_{\mathbb{A}\mathrm{gt}}^u$ is the set of uniform strategy profiles. Now, the requirement that agents from A should only use uniform strategies can be captured by the strategic constraint $\eta = \langle \Sigma^u, A \rangle$.

Consider CEGS M_2 from Example 2. For that model, the requirement that the only agent sticks to executable strategies can be captured by the constraint $\eta = \langle \{[q_0 \mapsto \alpha, q_1 \mapsto \alpha, q_2 \mapsto \alpha], [q_0 \mapsto \alpha, q_1 \mapsto \beta, q_2 \mapsto \beta]\}, \{a\} \rangle$.

The semantics of knowledge under strategic constraints is defined in a straightforward way: agents know that φ under η iff φ holds under η in every indistinguishable state. Note that this implicitly implies that the actual strategic constraint is common knowledge among the agents.

$M, q, \eta \models K_a\varphi$ iff $M, q', \eta \models \varphi$ for every q' such that $q \sim_a q'$.

$M, q, \eta \models \mathcal{K}_A\varphi$ iff $M, q', \eta \models \varphi$ for every q' such that $q \sim_A^{\mathcal{K}} q'$ (where $\mathcal{K} = C, E, D$).

We observe that strategic constraints influence only properties that involve strategic modalities $\langle\langle A \rangle\rangle$ (Proposition 5). Moreover, for a constraint $\eta = \langle \Upsilon, B \rangle$, only B's part of the profiles from Υ matters, as the following proposition shows (Proposition 6). Finally, properties of the original ATL can be seen as properties under a constraint that applies to nobody (Proposition 7).

Proposition 5 *Let η_1, η_2 be strategic constraints, and let φ be a formula that contains no cooperation modalities $\langle\langle A \rangle\rangle$. Then, for every M, q, we have $M, q, \eta_1 \models \varphi$ iff $M, q, \eta_2 \models \varphi$.*

Proposition 6 *Let $\Upsilon_1, \Upsilon_2 \subseteq \Sigma$ such that $\Upsilon_1[B] = \Upsilon_2[B]$. Then, for every M, q, φ, we have $M, q, \langle \Upsilon_1, B \rangle \models \varphi$ iff $M, q, \langle \Upsilon_2, B \rangle \models \varphi$.*

Proposition 7 *$M, q \models \varphi$ iff $M, q, \langle \Upsilon, \emptyset \rangle \models \varphi$.*

4 Translating Knowledge to Strategic Ability

ATL is trivially embedded into ATEL since all ATL formulae are also ATEL formulae. Moreover, every concurrent game structure can be extended to a concurrent epistemic game structure by defining all epistemic relations to be equalities, i.e. all agents have no uncertainty about the current state of the system. Finding an interpretation the other way is more involved. We will first construct a satisfaction-preserving interpretation of the fragment of ATEL without distributed knowledge (we will call it ATEL$_{CE}$), and then we will show how it can be extended to the whole ATEL, though at the expense of some increase of the model size.

The interpretation we discuss here is an update of that proposed in [7, 9]. Two things are changed. First, we slightly change the transformation of models so that, after visiting an "epistemic" state, the system *always* returns immediately to its corresponding "action" state. In consequence, it is possible to define the translation of formulae without exponential blowup in their length. Second, we show that the translation is also correct when we add constraints on the behavior of agents.

The original construction was inspired by [16], in which a propositional variant of the BDI logic [15] had been proved to be subsumed by propositional μ-calculus. Similar translations are well known within modal logics community, including translation of deontic logic into Propositional Dynamic Logic [14], translation of dynamic epistemic logic without common knowledge into epistemic logic [5] etc. In all these cases multi-modal logics are concerned, which makes it possible to "simulate" modalities of one kind with modalities of another kind. A work particularly close to ours is [20], where a (partial) reduction of ATEL model checking to ATL model checking was presented for a limited subclass of CEGS.

4.1 Idea of the Translation

ATEL consists of two orthogonal layers. The first one, inherited from ATL, refers to what agents can achieve in temporal perspective, and is underpinned by the structure defined via transition function o. The other layer is the epistemic component, reflected by epistemic accessibility relations. Our idea of the translation is to leave the original temporal structure intact, while extending it with additional transitions to "simulate" epistemic accessibility links. The "simulation" is achieved through adding new "epistemic" agents, who can enforce transitions to special "epistemic" copies of the original states (called "action" states in the extended model). The "action" and "epistemic" states form separate strata in the resulting model, and are labeled accordingly to distinguish transitions that implement different modalities.

Note: unlike in [7, 9], the "epistemic" states are not faithful copies of the corresponding "action" states. They copy neither the original valuation of propositions, nor the outgoing transitions. Instead, each epistemic state copies only the "epistemic" transitions of the original state, plus one transition that leads directly to the corresponding "action" state.

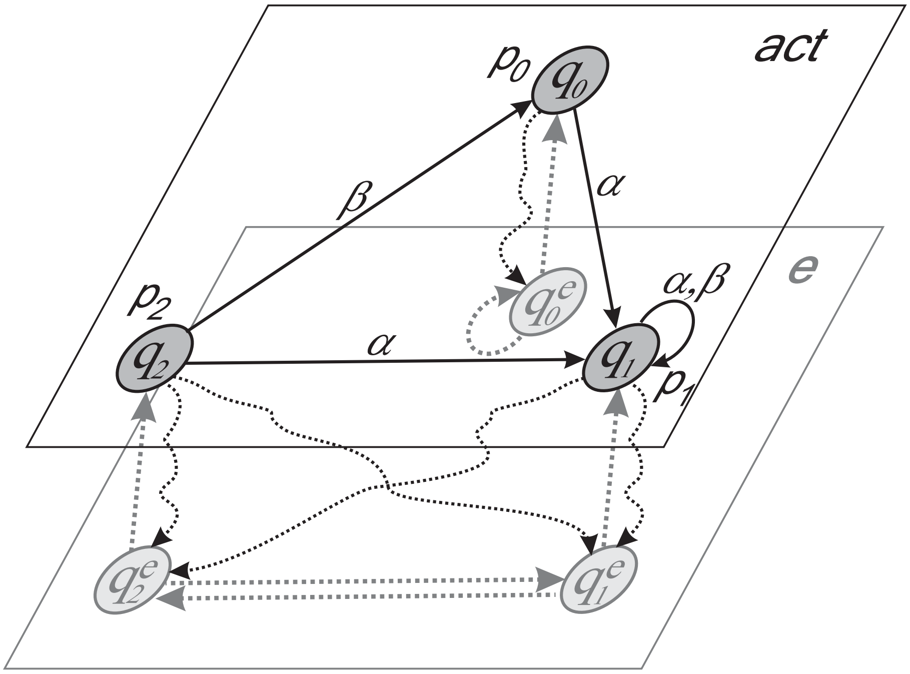

Figure 3: New model: "action" vs. "epistemic" states, and "action" vs. "epistemic" transitions.

The interpretation consists of two independent parts: a transformation of models and a translation of formulae. First, we propose a construction that transforms every concurrent epistemic game structure M for a set of agents $\{1, ..., k\}$, into a (pure) concurrent game structure M' over a set of agents $\{1, ..., k, e_1, ..., e_k\}$. Agents $1, ..., k$ are the original agents from M (we will call them "real agents"). Agents $e_1, ..., e_k$ are "epistemic doubles" of the real agents: the role of e_i is to "point out" the states that were epistemically indistinguishable from the current state for agent i in M. In order to distinguish transitions referring to different modalities, we introduce additional states in model M'. States $q_1^{\mathsf{e_i}}, ..., q_n^{\mathsf{e_i}}$ satisfy new proposition $\mathsf{e_i}$ added to enable identifying moves of epistemic agent e_i. Moreover, epistemic state $q^{\mathsf{e_i}}$ has the same "epistemic" transitions as q (leading to epistemic copies of states indistinguishable from q), plus one outgoing transition leading to the corresponding "action" state q. The original states $q_1, ..., q_n$ are still in M' to represent targets of "action" moves of the real agents $1, ..., k$. We will use a new proposition act to label these states. Now, the type of a transition can be recognized by the label of its target state. The idea of the transformation is depicted in Figure 3.

Defining the transition function o so that both epistemic and "action" transitions can occur is the trickiest part of the construction. We achieve this by giving priority to the epistemic agents' decisions. Every epistemic agent can choose to be "passive" and let the others decide upon the next move, or may try to effect an "epistemic" move. The resulting transition leads to the state selected by the *first* non-passive epistemic agent. If all the epistemic agents decided to be passive, the "action" transition chosen by the real agents follows. Again, "epistemic" states are given a special treatment, as the real agents have no substantial choice there. Thus, if all the epistemic agents decide to be passive at an "epistemic" state, the system proceeds to the corresponding "action" state.

With the above construction in mind, ATEL formulae can be translated to ATL according to the following scheme:

- $K_i\varphi$ can be rephrased as $\neg\langle\langle e_1, ..., e_i\rangle\rangle \bigcirc (\mathsf{e_i} \wedge \langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg\varphi))$: the epistemic moves to agent e_i's epistemic states do not lead to a state where φ fails (more precisely: where φ fails in the corresponding "action" state). Note that player e_i can select a state of his if, and only if, players $e_1, ..., e_{i-1}$ are passive (hence their presence in the cooperation modality). Note also that $K_i\varphi$ can be as well translated as $\neg\langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{e_i} \wedge \langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg\varphi))$ or $\neg\langle\langle 1, ..., k, e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{e_i} \wedge \langle\langle 1, ..., k, e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg\varphi))$: when e_i decides to be active, choices from $1, ..., k$ and $e_{i+1}, ..., e_k$ are irrelevant.

- $\langle\langle A\rangle\rangle \bigcirc \varphi$ becomes $\langle\langle A \cup \{e_1, ..., e_k\}\rangle\rangle \bigcirc (\mathsf{act} \wedge \varphi)$ in a similar way.

- Translation of the other temporal operators is now more straightforward than in [7, 9]: $\langle\langle A\rangle\rangle \Box\varphi$ can be rephrased as $\langle\langle A \cup \{e_1, ..., e_k\}\rangle\rangle \Box(\mathsf{act} \wedge \varphi)$, and $\langle\langle A\rangle\rangle \varphi\,\mathcal{U}\,\psi$ becomes $\langle\langle A \cup \{e_1, ..., e_k\}\rangle\rangle (\mathsf{act} \wedge \varphi)\,\mathcal{U}\,(\mathsf{act} \wedge \psi)$. This is possible because the construction of epistemic states (and the translation of K_a) ensures that strategic (sub)formulae will be always evaluated in "action" states. We observe that the new translation of $\Box$ and $\mathcal{U}$ does not involve exponential increase in the length of formulae (contrary to the construction from [7, 9]).

- Translation of mutual knowledge (E_A) is analogous to the individual knowledge case. Translation of common knowledge refers to the definition of relation $\sim_A^C$ as the transitive closure of relations $\sim_i$ for $i \in A$: $C_A\varphi$ means that all the (finite) sequences of appropriate epistemic transitions must end up in a state where φ is true.

The only operator that does not seem to lend itself to a translation according to the above scheme is the distributed knowledge operator D, for which we seem to need more "auxiliary" agents. Thus, we will begin with presenting details of our interpretation for ATEL_{CE} – a reduced version of

ATEL that includes only common knowledge and "everybody knows" operators for group epistemics. Section 4.3 shows how to modify the translation to include distributed knowledge as well.

4.2 Interpreting Models and Formulae of ATEL$_{CE}$ into ATL

4.2.1 Transforming Models

Given a concurrent epistemic game structure $M = \langle \mathbb{A}\mathrm{gt}, St, \Pi, \pi, Act, d, o, \sim_1, ..., \sim_k \rangle$, we construct a new concurrent game structure $M' = \langle \mathbb{A}\mathrm{gt}', St', \Pi', \pi', Act', d', o' \rangle$ as follows:

- $\mathbb{A}\mathrm{gt}' = \mathbb{A}\mathrm{gt} \cup \mathbb{A}\mathrm{gt}^e$, where $\mathbb{A}\mathrm{gt}^e = \{e_1, ..., e_k\}$ is the set of epistemic agents;
- $St' = St \cup St^{\mathsf{e}_1} \cup ... \cup St^{\mathsf{e}_k}$, where $St^{\mathsf{e}_i} = \{q^{\mathsf{e}_i} \mid q \in St\}$. We assume that $St, St^{\mathsf{e}_1}, ..., St^{\mathsf{e}_k}$ are pairwise disjoint. Further we will be using the more general notation $S^{\mathsf{e}_i} = \{q^{\mathsf{e}_i} \mid q \in S\}$ for any $S \subseteq St$;
- $\Pi' = \Pi \cup \{\mathsf{act}, \mathsf{e}_1, ..., \mathsf{e}_k\}$;
- $\pi'(p) = \pi(p)$ for every proposition $p \in \Pi$. Moreover, $\pi'(\mathsf{act}) = St$ and $\pi'(\mathsf{e}_i) = St^{\mathsf{e}_i}$;
- $Act' = Act \cup St \cup \{pass\}$: the new model M' contains the original actions from M, plus epistemic actions (pointing indistinguishable states), and the "do nothing" action $pass$;
- $d'_a(q) = d_a(q)$ for $a \in \mathbb{A}\mathrm{gt}$, $q \in St$: choices of the real agents do not change in the original ("action") states;
- $d'_a(q) = \{pass\}$ for all $a \in \mathbb{A}\mathrm{gt}$, $q \in St' \setminus St$: the real agents behave automatically in the new (epistemic) states;
- $d'_{e_i}(q) = \mathrm{img}(q, \sim_i) \cup \{pass\}$ for $q \in St'$: an epistemic agent can point out an indistinguishable state or choose to remain passive;
- the new transition function is defined as follows:

$$o'(q, \alpha_1, ..., \alpha_k, \alpha_{e_1}, ..., \alpha_{e_k}) = \begin{cases} o(q, \alpha_1, ..., \alpha_k) & \text{if } q \in St \text{ and} \\ & \alpha_{e_1} = ... = \alpha_{e_k} = pass \\ q_0 & \text{if } q = q_0^{\mathsf{e}_i} \in St^{\mathsf{e}_i} \text{and} \\ & \alpha_{e_1} = ... = \alpha_{e_k} = pass \\ (\alpha_{e_i})^{e_i} & \text{if } e_i \text{ is the first active} \\ & \text{epistemic agent.} \end{cases}$$

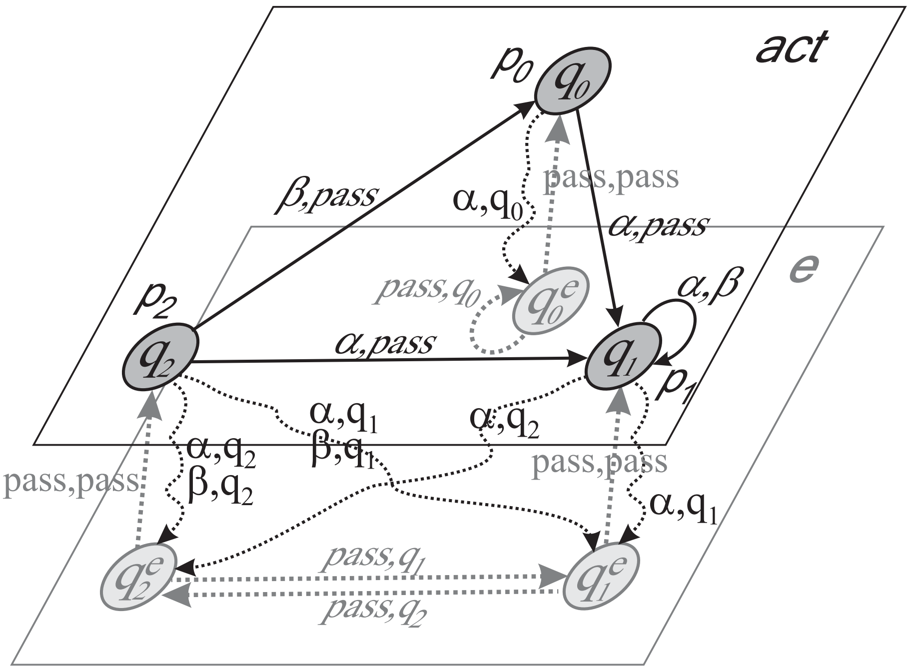

Figure 4: Reconstruction for the concurrent epistemic game structure from Figure 2.

We assume that all the epistemic agents from $\mathbb{A}\mathrm{gt}^e$, states from $St^{\mathsf{e}_1} \cup \ldots \cup St^{\mathsf{e}_k}$, and propositions from $\{\mathsf{act}, \mathsf{e}_1, \ldots, \mathsf{e}_k\}$, are new and were absent in the original model M.

Example 3 The transformation of the simple CEGS from Figure 2 is shown in Figure 4. ■

4.2.2 Translation of Formulae

Now, we define a translation of formulae from ATEL_{CE} to ATL corresponding to the above transformation of models:

$$
\begin{aligned}
tr(p) &= p, \qquad \text{for } p \in \Pi \\
tr(\neg\varphi) &= \neg tr(\varphi) \\
tr(\varphi \vee \psi) &= tr(\varphi) \vee tr(\psi) \\
tr(\langle\!\langle A \rangle\!\rangle \bigcirc \varphi) &= \langle\!\langle A \cup \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \bigcirc (\mathsf{act} \wedge tr(\varphi)) \\
tr(\langle\!\langle A \rangle\!\rangle \Box \varphi) &= \langle\!\langle A \cup \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \Box (\mathsf{act} \wedge tr(\varphi)) \\
tr(\langle\!\langle A \rangle\!\rangle \varphi \mathcal{U} \psi) &= \langle\!\langle A \cup \mathbb{A}\mathrm{gt}^e \rangle\!\rangle (\mathsf{act} \wedge tr(\varphi)) \mathcal{U} (\mathsf{act} \wedge tr(\psi)) \\
tr(K_i \varphi) &= \neg \langle\!\langle e_1, \dots, e_i \rangle\!\rangle \bigcirc \big(\mathsf{e_i} \wedge \langle\!\langle e_1, \dots, e_k \rangle\!\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\varphi))\big) \\
tr(E_A \varphi) &= \neg \langle\!\langle \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \bigcirc \Big(\big(\bigvee_{a_i \in A} \mathsf{e_i}\big) \wedge \langle\!\langle e_1, \dots, e_k \rangle\!\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\varphi))\Big) \\
tr(C_A \varphi) &= \neg \langle\!\langle \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \bigcirc \langle\!\langle \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \\
&\qquad \big(\bigvee_{a_i \in A} \mathsf{e_i}\big) \mathcal{U} \Big(\big(\bigvee_{a_i \in A} \mathsf{e_i}\big) \wedge \langle\!\langle e_1, \dots, e_k \rangle\!\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\varphi))\Big).
\end{aligned}
$$

4.2.3 Extending Strategic Constraints

Given a strategic constraint $\eta = \langle \Upsilon, B \rangle$ in M, we must extend it to match the type of constraints in M' (because M' includes more agents than M, and in consequence the elements of Υ, which are full strategy profiles in M, are only partial profiles in M'). This can be done in many ways; here, we explicitly assume that the additional (epistemic) agents can use any strategies they like. The new constraint must apply to the agents from B, plus (possibly) to some of the new agents from $\mathbb{A}\mathrm{gt}^e$. That is, agents from B are constrained in the same way as before, agents from $\mathbb{A}\mathrm{gt} \setminus B$ are unconstrained in the same way as before, and the new agents can be put under constraints or not – but even if they are, they can play any available strategy.[5]

Definition 5 *Let $\eta = \langle \Upsilon, B \rangle$ be a strategic constraint in concurrent epistemic game structure M, and let M' be the concurrent game structure obtained from M by the construction presented in Section 4.2.1. We say that constraint $\eta' = \langle \Upsilon', B' \rangle$* extends *$\eta$ in M' iff:*

1. $\Upsilon' = \Upsilon \times \Sigma_{\mathbb{A}\mathrm{gt}^e}$, *and*
2. $B \subseteq B' \subseteq B \cup \mathbb{A}\mathrm{gt}^e$.

[5] We recall that the assumption that a player plays a memoryless strategy is itself a restriction on the agent's behavior.

4.2.4 Properties of the Translation

First, we show that the translation is correct; then, we discuss the complexity of the resulting models and formulae.

Theorem 8 *Let φ be a formula of* ATEL_{CE}*, M be a* CEGS*, and $q \in St$ an "action" state in M. Furthermore, let η be a behavioral constraint in M, and let η' extend η in M'. Then, $M, q, \eta \models \varphi$ iff $M', q, \eta' \models tr(\varphi)$.*

Proof. The proof follows by structural induction on φ. We will show that the construction preserves the truth value of φ for four cases: $\varphi \equiv p$, $\varphi \equiv \neg\psi$, $\varphi \equiv \langle\langle A\rangle\rangle \Box\psi$ and $\varphi \equiv K_a\psi$. An interested reader can tackle the other cases in an analogous way.

case $\varphi \equiv p$**,** $\mathbf{ATEL}_{CE} \Leftrightarrow$ **ATL.** $M, q, \eta \models p$ iff $q \in \pi(p)$ iff $q \in \pi'(p)$ iff $M', q, \eta' \models p$ iff $M', q, \eta' \models tr(p)$.

case $\varphi \equiv \neg\psi$**,** $\mathbf{ATEL}_{CE} \Leftrightarrow$ **ATL.** $M, q, \eta \models \neg\psi$ iff $M, q, \eta \not\models tr(\psi)$ iff (by induction) $M', q, \eta' \not\models tr(\psi)$ iff $M', q, \eta' \models \neg tr(\psi)$ iff $M', q, \eta' \models tr(\neg\psi)$.

case $\varphi \equiv \langle\langle A\rangle\rangle \Box\psi$**,** $\mathbf{ATEL}_{CE} \Rightarrow$ **ATL.** Let $M, q, \eta \models \langle\langle A\rangle\rangle \Box\psi$, then A have a collective strategy s_A, consistent with η, such that for every $\lambda \in out_M(q, s_A, \eta)$ and $i \geq 0$ we have $M, \lambda[i], \eta \models \psi$ (*). Consider strategy $t_{A\cup\mathbb{A}\mathrm{gt}^e}$ in the new model M', such that $t^a_{A\cup\mathbb{A}\mathrm{gt}^e}(q) = s^a_A(q)$ for all $a \in A, q \in St$, and $t^a_{A\cup\mathbb{A}\mathrm{gt}^e}(q) = pass$ for all $a \in \mathbb{A}\mathrm{gt}^e$. In other words, $t_{A\cup\mathbb{A}\mathrm{gt}^e}$ is a strategy according to which the agents from A do exactly the same as in the original strategy s_A for all the "action" states from St (and pass elsewhere), while the epistemic agents remain passive all the time. Note that $t_{A\cup\mathbb{A}\mathrm{gt}^e}$ is consistent with η' (as $t_{A\cup\mathbb{A}\mathrm{gt}^e}[B] = s_{A\cap B}$ must occur in $\Upsilon'[B] = \Upsilon[B]$, and all the strategies of $\mathbb{A}\mathrm{gt}^e$ are allowed in Υ' in all possible combinations).[6] Since all the epistemic agents pass, every $\lambda \in out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta)$ includes only "action" states from St (**), and thus every such λ is also a path in M. As the agents from A make the same choices in $s_A, t_{A\cup\mathbb{A}\mathrm{gt}^e}$, and η, η' agree on their parts that are relevant for M, we obtain that $out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta') = out_M(q, s_A, \eta)$. By (*): for every $\lambda \in out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta')$ and $i \geq 0$ we have $M, \lambda[i], \eta \models \psi$. By induction we get that for every $\lambda \in out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta')$ and $i \geq 0$ we have $M', \lambda[i], \eta' \models tr(\psi)$. By (**), also $M', \lambda[i], \eta' \models \mathsf{act}$ for all such λ, i. In consequence, $M', q, \eta' \models \langle\langle A \cup \mathbb{A}\mathrm{gt}^e\rangle\rangle \Box(\mathsf{act} \wedge tr(\psi))$.

case $\varphi \equiv \langle\langle A\rangle\rangle \Box\psi$**, ATL** $\Rightarrow \mathbf{ATEL}_{CE}$**.** Let $M', q, \eta' \models \langle\langle A \cup \mathbb{A}\mathrm{gt}^e\rangle\rangle \Box(\mathsf{act} \wedge tr(\psi))$. Then, there is $t_{A\cup\mathbb{A}\mathrm{gt}^e}$, consistent with η', such that $M', \lambda[i], \eta' \models \mathsf{act} \wedge tr(\psi)$ for all $\lambda \in out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta')$ and $i \geq 0$. By the fact that $M', \lambda[i], \eta' \models \mathsf{act}$, we can infer that all such paths λ consist only of "action" states,

[6] We recall that $B \subseteq \mathbb{A}\mathrm{gt}$.

and thus they are also paths in M. By the induction hypothesis, it follows that $M, \lambda[i], \eta \models \psi$ for all $\lambda \in out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta')$ and $i \geq 0$. Now, let $s_A = t_{A\cup\mathbb{A}\mathrm{gt}^e}[A]$. We observe that s_A is a strategy in M as well; moreover, s_A is consistent with η, as it is a part of an η'-consistent strategy, and η, η' agree on their parts referring to the agents from $\mathbb{A}\mathrm{gt}$. Finally, the epistemic agents in $t_{A\cup\mathbb{A}\mathrm{gt}^e}$ always remain passive, which means that only the choices of A are relevant for the actual course of action, so $out_{M'}(q, t_{A\cup\mathbb{A}\mathrm{gt}^e}, \eta') = out_M(q, s_A, \eta)$. Summing it up, we get that there is s_A, consistent with η, such that $M, \lambda[i], \eta \models \psi$ for all $\lambda \in out_M(q, s_A, \eta)$ and $i \geq 0$. In consequence, $M, q, \eta \models \langle\langle A\rangle\rangle \Box\psi$.

case $\varphi \equiv K_i\psi$**, ¬ATL ⇒ ¬ATEL**$_{CE}$**.** Let $M', q, \eta \models \langle\langle e_1, ..., e_i\rangle\rangle \bigcirc \big(\mathsf{e_i} \wedge \langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\psi))\big)$. Then, there is a collective choice $\langle \alpha_{e_1}, ..., \alpha_{e_i}\rangle$ such that $\mathsf{e_i}$ (*) and $\langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\psi))$ (**) hold in the next moment (for every response from the rest of agents).[7] By (*) we have that $\alpha_{e_1} = ... = \alpha_{e_{i-1}} = pass$ and $\alpha_{e_i} = q_x$ with the epistemic link $q \sim_a q_x$ present in the original CEGS M (note also that the choices of the other agents are irrelevant). Moreover, the next state is going to be $q_x^{e_i}$. By (**) we have that $M', q_x^{e_i}, \eta' \models \langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\psi))$. The only choice of $\{e_1, ..., e_k\}$ at $q_x^{e_i}$ that guarantees that act holds next is $\alpha_{e_1} = ... = \alpha_{e_k} = pass$, and the subsequent state will be q_x. In consequence, it must hold that $M', q_x, \eta' \models \neg tr(\psi)$, i.e, $M', q_x, \eta' \not\models tr(\psi)$. Note that q_x is an "action" state, so by induction we get that $M, q_x, \eta \not\models \psi$. But this means that $M, q, \eta \models \neg K\psi$, as $q \sim_a q_x$.

case $\varphi \equiv K_i\psi$**, ¬ATEL**$_{CE}$ **⇒ ¬ATL.** We have $M, q, \eta \not\models K_a\psi$, so there is q_x with $q \sim_a q_x$ and $M, q_x, \eta \not\models \psi$. By induction, we get $M', q_x, \eta' \not\models tr(\psi)$. As q_x is an action state, it holds that $M', q_x, \eta' \models \mathsf{act} \wedge \neg tr(\psi)$. Consider the "epistemic" state $q_x^{e_i}$ in M', and the collective choice $\alpha_{e_1} = ... = \alpha_{e_k} = pass$ at that state. First, the choice is consistent with η' (all choices from the epistemic agents are); second, it enforces a transition to q_x, so $M', q_x^{e_i}, \eta' \models \mathsf{e_i} \wedge \langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\psi))$. But the epistemic agents $\{e_1, ..., e_i\}$ have a collective choice at q that enforces a transition to $q_x^{e_i}$, namely $\alpha_{e_1} = ... = \alpha_{e_{i-1}} = pass, \alpha_{e_i} = q_x$. Thus, we get $M', q, \eta' \models \langle\langle e_1, ..., e_i\rangle\rangle \bigcirc \big(\mathsf{e_i} \wedge \langle\langle e_1, ..., e_k\rangle\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\psi))\big)$, which concludes the proof.

■

Note that the construction used above to interpret ATEL$_{CE}$ in ATL has several nice complexity properties. In the following list, k denotes the number of agents, p the number of propositions, n the number of states, m the number of transitions, and $\overline{m}$ the number of epistemic links in the original CEGS

[7] Note that *every* collective strategy of $\{e_1, ..., e_i\}$ is consistent with η' by definition.

M. Likewise, k', p', n', m' denote the number of agents, propositions, states and transitions in the resulting CGS M'.

- The vocabulary (set of propositions Π) and the set of agents only increase linearly: $p' = p + k + 1 = O(p + k)$ and $k' = 2k = O(k)$.
- The set of states in an ATEL-model grows linearly, too: $n' = (k + 1)n = O(kn)$.
- We have $m' = m + k(\overline{m} + 1) = O(m + k\overline{m})$ transitions in M' (m "action" transitions and $\overline{m}$ epistemic transitions from "action" states, plus $\overline{m} + 1$ transitions from each "epistemic" state).
- The length of formulae also increases linearly: $l \leq l' \leq l(8+5k) = O(kl)$.

Note also that the transformation of models and formulae is straightforward, and in consequence its complexity is no worse than the corresponding complexities of the resulting structures.

4.3 Handling Distributed Knowledge

In order to interpret the full ATEL we modify the construction from Section 4.2 by introducing additional epistemic agents (and states) indexed with coalitions which occur with a distributed knowledge operator:

- $\mathbb{A}\mathrm{gt}^e = \{e_i \mid i \in \mathbb{A}\mathrm{gt}\} \cup \{e_A \mid D_A \in \varphi\}$;
- $St' = St \cup \bigcup_{i \in \mathbb{A}\mathrm{gt}} St^{\mathsf{e_i}} \cup \bigcup_{D_A \in \varphi} St^{\mathsf{e_A}}$.

Accordingly, we extend the language with new propositions $\{\mathsf{e_i} \mid i \in \mathbb{A}\mathrm{gt}\}$ and $\{\mathsf{e_A} \mid D_A \in \varphi\}$. The choices of collective epistemic agents e_A refer to the (epistemic copies of) states accessible via distributed knowledge relations:

- $d'_{e_A}(q) = \{pass\} \cup img(q, \sim_A^D)^{\mathsf{e_A}}$.

The new transition function extends the one from Section 4.2 with choices of agents e_A (putting them in any predefined order, e.g. alphabetical order):

$$o'(q, \alpha_1, ..., \alpha_k, \alpha_{e_1}, ..., \alpha_{e_k}, ..., \alpha_{e_A}, ...) = \begin{cases} o(q, \alpha_1, ..., \alpha_k) & \text{if } q \in St \text{ and} \\ & \alpha_a = pass \text{ for all } a \in \mathbb{A}\mathrm{gt}^e \\ q_0 & \text{if } q = q_0^{\mathsf{e_i}} \in St^{\mathsf{e_i}} \text{ and} \\ & \alpha_a = pass \text{ for all } a \in \mathbb{A}\mathrm{gt}^e \\ (\alpha_{e_a})^{e_a} & \text{if } e_a \text{ is the first active} \\ & \text{epistemic agent.} \end{cases}$$

The translation of formulae for all operators of ATEL$_{CE}$ remains the same as well, and the translation of D_A is:

$$tr(D_A\varphi) \quad = \quad \neg\langle\!\langle \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \bigcirc \big(\mathsf{e_i} \wedge \langle\!\langle \mathbb{A}\mathrm{gt}^e \rangle\!\rangle \bigcirc (\mathsf{act} \wedge \neg tr(\varphi))\big).$$

The following result can now be proved similarly to Theorem 8.

Theorem 9 *Let φ be a formula of* ATEL*, M be a* CEGS*, and $q \in St$ an "action" state in M. Furthermore, let η be a behavioral constraint in M, and let η' extend η in M'. Then, $M, q, \eta \models \varphi$ iff $M', q, \eta' \models tr(\varphi)$.*

This construction, too, does not involve any substantial increase of complexity:

- $p' = O(p + k + l)$,
- $k' = O(k + l)$,
- $n' = O(kn + ln)$,
- $m' = O(m + k\overline{m} + l\overline{m})$,
- $l' = O(kl)$.

Still, it has one disadvantage when compared to the construction from Section 4.2: in the previous construction (for ATEL$_{CE}$), models and formulae could be translated independently; in our construction for full ATEL, the transformation of a model depends on the formula which is going to be model-checked. Thus, it is not possible any more to "pre-compile" a given CEGS in advance, and then model-check on the fly any formulae that will become relevant.

5 Conclusions

In this report, we propose an update of the reduction scheme that was presented in [7, 9]. The original reduction allowed to get rid of epistemic operators by translating them to cooperation modalities of ATL which made use of additional "epistemic" agents. The new version has two new features. First, we avoid the exponential blowup of formulae, which was to some extent present in the original reduction. Second, we show that the reduction is valid also if we specify *strategic constraints* which restrict collective strategies that some (or all) agents are allowed to use. Thus, the applicability of the new reduction scheme goes well beyond ATEL (i.e., perfect information ATL + knowledge operators). We can use the scheme to translate knowledge to strategic ability for agents playing under imperfect information (like in ATLir from [17]), acting in the presence of social norms [18], or choosing only rational play [10]. It seems that many other extensions of alternating-time temporal logic should submit to the reduction, too.

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